Factoring

Numbers have factors:

$$\underbrace{2}_{\text{Factor}} \times \underbrace{3}_{\text{Factor}} = 6$$

And expressions (like $\mathbf{x^2 + 4x + 3}$) also have factors:

$$\underbrace{(x+3)}_{\text{Factor}}\underbrace{(x+1)}_{\text{Factor}} = x^2 + 4x + 3$$

Factoring

Factoring (called "**Factorising**" in the UK) is the process of **finding the factors**:

> Factoring: Finding what to multiply together to get an expression.

It is like "splitting" an expression into a multiplication of simpler expressions.

Example: factor $2y + 6$

Both 2y and 6 have a common factor of 2:

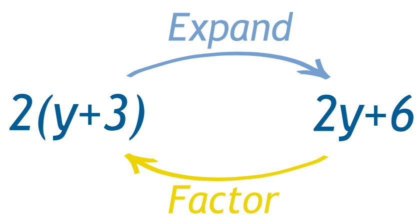
- $2y$ is $2 \times y$
- 6 is 2×3

So we can factor the whole expression into:

$$2y+6 = 2(y+3)$$

So **2y+6** has been "factored into" **2** and **y+3**

Factoring is also the opposite of [Expanding]:



Common Factor

In the previous example we saw that 2y and 6 had a common factor of **2**

But to do the job properly we need the **highest common factor**, including any variables

Example: factor $3y^2 + 12y$

First, 3 and 12 have a common factor of 3.

So we could have:

$$3y^2 + 12y = 3(y^2 + 4y)$$

But we can do better!

$3y^2$ and $12y$ also share the variable y.

Together that makes $3y$:

- $3y^2$ is $3y \times y$
- $12y$ is $3y \times 4$

So we can factor the whole expression into:

$$3y^2 + 12y = 3y(y + 4)$$

Check: **$3y(y+4) = 3y \times y + 3y \times 4 = 3y^2 + 12y$**

More Complicated Factoring

Because we have to figure **what got multiplied** to produce the expression we are given!

? x ? = 

It is like trying to find which ingredients
went into a cake to make it so delicious.
It can be hard to figure out!

Experience Helps

With more experience factoring becomes easier.

Example: Factor $\mathbf{4x^2 - 9}$

Hmmm... there don't seem to be any common factors.

But knowing the Special Binomial Products gives us a clue called the **"difference of squares"**:

$$4x^2 - 9$$

$$(2x)^2 \; - \; 3^2$$

Because $\mathbf{4x^2}$ is $\mathbf{(2x)^2}$, and **9** is $\mathbf{(3)^2}$,

So we have:

$$4x^2 - 9 \;=\; (2x)^2 - (3)^2$$

And that can be produced by the difference of squares formula:

$$(a+b)(a-b) \;=\; a^2 - b^2$$

Where **a** is 2x, and **b** is 3.

So let us try doing that:

$$(2x+3)(2x-3) \;=\; (2x)^2 - (3)^2 \;=\; 4x^2 - 9$$

Yes!

So the factors of $\mathbf{4x^2 - 9}$ are **(2x+3)** and **(2x−3)**:

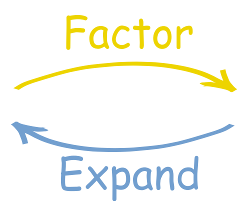
Answer: $4x^2 - 9 \;=\; (2x+3)(2x-3)$

How can you learn to do that? By getting lots of practice, and knowing "Identities"!

Remember these Identities

Here is a list of common "Identities" (including the **"difference of squares"** used above).

It is worth remembering these, as they can make factoring easier.



$$
\begin{aligned}
a^2 - b^2 &= (a+b)(a-b) \\
a^2 + 2ab + b^2 &= (a+b)(a+b) \\
a^2 - 2ab + b^2 &= (a-b)(a-b)
\end{aligned}
$$

There are many more like those, but those are the most useful ones.

Advice

The factored form is usually best.

When trying to factor, follow these steps:

- "Factor out" any common terms

- See if it fits any of the identities, plus any more you may know
- Keep going till you can't factor any more

More Examples

Experience does help, so here are more examples to help you on the way:

Example: $w^4 - 16$

An exponent of 4? Maybe we could try an exponent of 2:

$$w^4 - 16 = (w^2)^2 - 4^2$$

And it is now the difference of squares

$$w^4 - 16 = (w^2 + 4)(w^2 - 4)$$

And "$(w^2 - 4)$" is another difference of squares

$$w^4 - 16 = (w^2 + 4)(w + 2)(w - 2)$$

That is as far as we can go (unless we use imaginary numbers)

Example: $3u^4 - 24uv^3$

Remove common factor "3u":

$$3u^4 - 24uv^3 = 3u(u^3 - 8v^3)$$

Then a difference of cubes:

$$3u^4 - 24uv^3 = 3u(u^3 - (2v)^3)$$

$$= 3u(u-2v)(u^2+2uv+4v^2)$$

That is as far as we can go.

Example: $z^3 - z^2 - 9z + 9$

Try factoring the first two and second two separately:

$$z^2(z-1) - 9(z-1)$$

Wow, $(z\text{-}1)$ is on both, so let us use that:

$$(z^2-9)(z-1)$$

And z^2-9 is a difference of squares

$$(z-3)(z+3)(z-1)$$

That is as far as we can go.