

CHAPTER 6

Ratios, Rates, and Proportions

6-1 Ratios and Rates

A **ratio** is a comparison of two quantities by division.

The ratio of a to b can be written in three different ways: a to b , $a:b$, and $\frac{a}{b}$.

If two quantities are in the ratio of a to b , then the two numbers can be expressed as ax and bx , in which x is a positive integer.

A **rate** is a ratio of two measurements having different units of measure. For example, a price of \$2.59 per gallon of gasoline, an income of \$750 in 3 days, and a speed of 60 miles per hour are all rates.

A **unit rate** is a rate that has a denominator of 1. Some examples of unit rates are defined as follows.

$$\text{Unit Price} = \frac{\text{Price of Package}}{\text{Number of Units in the Package}}$$

$$\text{Gas Mileage} = \frac{\text{Number of Miles Traveled}}{\text{Number of Gallons of Gas Used}}$$

$$\text{Speed (Miles per Hour)} = \frac{\text{Number of Miles Traveled}}{\text{Number of Hours}}$$

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

- Example 1 □ Express each ratio as a unit rate.
- 1360 grams of coffee cost \$17.68. What is the unit price of the coffee?
 - A car travels 322 miles on 11.5 gallons of gas. What is the car's gas mileage?
 - A driver traveled $485\frac{1}{3}$ miles in $8\frac{2}{3}$ hours. What is his speed?
 - A volume of 46 cm^3 of silver has a mass of 483 grams. What is the density of silver?

Solution □

- $$\frac{\$17.68}{1360 \text{ grams}} = \frac{1768 \text{ cents}}{1360 \text{ grams}} = 1.3 \text{ cents / gram}$$
- $$\frac{322 \text{ mi}}{11.5 \text{ gal}} = 28 \text{ miles / gallon .}$$
- $$\frac{485\frac{1}{3} \text{ mi}}{8\frac{2}{3} \text{ hr}} = \frac{(485\frac{1}{3} \text{ mi}) \cdot 3}{(8\frac{2}{3} \text{ hr}) \cdot 3} = \frac{1456 \text{ mi}}{26 \text{ hr}} = 56 \text{ mph}$$
- $$\text{density} = \frac{\text{mass}}{\text{volume}} = \frac{483 \text{ grams}}{46 \text{ cm}^3} = 10.5 \text{ grams/cm}^3$$

- Example 2 □ 3 angles of a triangle are in the ratio of 3 : 5 : 7 . What is the measure of each angle?

Solution □ The measure of each angle of the triangle can be represented as $3x$, $5x$, and $7x$.

$3x + 5x + 7x = 180$	The angle sum in a triangle is 180.
$15x = 180$	Simplify.
$x = 12$	Simplify.

The measure of the 3 angles are $3x = 3 \cdot 12 = 36$, $5x = 5 \cdot 12 = 60$, and $7x = 7 \cdot 12 = 84$.

Exercises - Ratios and Rates

1

The ratio of $1\frac{3}{4}$ to $2\frac{1}{2}$ is equal to the ratio of 14 to what number?

- A) 18
- B) 20
- C) 22
- D) 24

2

The sum of two numbers is 14 and the ratio of the two numbers is -3 . What is the product of the two numbers?

- A) -105
- B) -119
- C) -133
- D) -147

3

If $2(x - y) = 3y$, what is the ratio $\frac{x}{y}$?

- A) $\frac{2}{5}$
- B) $\frac{4}{3}$
- C) $\frac{5}{2}$
- D) $\frac{8}{3}$

4

The ratio of length to width of a rectangular garden is $6 : 7$. If the perimeter of the rectangle is 78 meters, what is the area of the garden in square meters?

- A) 274
- B) 326
- C) 352
- D) 378

5

A car travels 218.5 miles on 9.5 gallons of gas. What is the car's gas mileage?

6

At a grocery store, 20 fl oz of brand *A* vitamin water is sold for \$0.95. What is the unit price of the vitamin water per ounce, to the nearest cents?

7

The density of aluminum is 2.7 grams per cm^3 . How many grams does 12 cm^3 of aluminum weigh?

6-2 Proportions

A **proportion** is an equation stating that two ratios are equal.

The proportions $\frac{a}{b} = \frac{c}{d}$ is read “ a is to b as c is to d .” The numbers a and d are called the **extremes** of the proportion. The numbers b and c are called the **means** of the proportion.

In a proportion, the product of the extremes is equal to the product of the means.

If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$. The products ad and bc are called the **cross products** of the proportion $\frac{a}{b} = \frac{c}{d}$.

Example 1 □ Determine whether each pair of ratios forms a proportion.

a. $\frac{0.4}{1.5}, \frac{1.6}{6}$

b. $\frac{12}{25}, \frac{7}{15}$

Solution □ a. $\frac{0.4}{1.5} \stackrel{?}{=} \frac{1.6}{6}$

Write a proportion.

$$\begin{aligned} 0.4 \times 6 &\stackrel{?}{=} 1.5 \times 1.6 \\ 2.4 &= 2.4 \end{aligned}$$

Find the cross products.
Simplify.

The cross products are equal, so the ratios form a proportion.

b. $\frac{12}{25} \stackrel{?}{=} \frac{7}{15}$

Write a proportion.

$$\begin{aligned} 12 \times 15 &\stackrel{?}{=} 25 \times 7 \\ 180 &\neq 175 \end{aligned}$$

Find the cross products.
Simplify.

The cross products are not equal, so the ratios do not form a proportion.

Example 2 □ Solve the proportion $\frac{3}{7} = \frac{6}{x-4}$.

Solution □ $\frac{3}{7} = \frac{6}{x-4}$

$$3(x-4) = 7(6)$$

Find the cross products.

$$3x - 12 = 42$$

Distributive Property

$$3x = 54$$

Add 12 to each side.

$$x = 18$$

Divide each side by 3.

Example 3 □ Carter's SUV requires 8 gallons of gasoline to travel 148 miles. How much gasoline, to the nearest gallon, will he need for a 500 mile trip?

Solution □ a. Let g = the number of gallons of gas needed for a 500 mile trip.

$$\begin{array}{lcl} \text{miles} & \rightarrow & \frac{148}{8} = \frac{500}{g} \leftarrow \text{miles} \\ \text{gallons} & \rightarrow & \end{array}$$

Write a proportion.

$$148g = 8 \times 500$$

Find the cross products.

$$g = \frac{8 \times 500}{148} \approx 27.03$$

Divide each side by 148.

Carter's needs 27 gallons of gas for a 500 mile trip.

Exercises - Proportions

1

On a map, 1 inch represents 5 miles. If a certain state is represented on a map by a rectangle 10 inches by 7.2 inches, what is the area of the state in square miles?

- A) 360 mi^2
- B) 720 mi^2
- C) $1,080 \text{ mi}^2$
- D) $1,800 \text{ mi}^2$

2

Together there are 754 students and teachers in the meeting. If the ratio of students to teachers is $27 : 2$, how many teachers are there?

- A) 46
- B) 52
- C) 58
- D) 64

3

Concrete is made by mixing cement, sand, and gravel in the ratio $5 : 9 : 13$. How much cement is needed to make 324 ft^3 of concrete?

- A) 54 ft^3
- B) 60 ft^3
- C) 84 ft^3
- D) 108 ft^3

4

If Andy drove 84 miles in 1 hour 45 minutes, how many miles can he drive in 5 hours?

5

A collection of quarters, dimes, and nickels is worth \$5.00. If the ratio of quarters to dimes to nickels is $2 : 4 : 7$, how many quarters are there?

6

If $\frac{5x}{3} = \frac{x+14}{2}$, what is the value of x ?

7

A trail mix contains raisin, peanut, and chocolate. The ratio of raisin to peanut is $2 : 3$ and the ratio of peanut to chocolate is $5 : 8$. What is the ratio of raisin to chocolate?

Answer Key

Section 6-1

1. B 2. D 3. C 4. D 5. 23
6. 5 7. 32.4

Section 6-2

1. D 2. B 3. B 4. 240 5. 8
6. 6 7. $\frac{5}{12}$

Section 6-3

1. A 2. D 3. B 4. B 5. C
6. D

Chapter 6 Practice Test

1. C 2. A 3. C 4. C 5. B
6. A 7. A 8. B 9. B 10. D
11. $\frac{25}{4}$ or 6.25 12. 3

Answers and Explanations**Section 6-1**

1. B

$$\frac{1\frac{3}{4}}{2\frac{1}{2}} = \frac{14}{x}$$

The ratio of $1\frac{3}{4}$ to $2\frac{1}{2}$ is equal to the ratio of 14 to x .

$$1\frac{3}{4} \cdot x = 14 \cdot 2\frac{1}{2}$$

Cross Products

$$\frac{7}{4}x = 14 \cdot \frac{5}{2}$$

Simplify.

$$\frac{7}{4}x = 35$$

Simplify.

$$\frac{4}{7} \cdot \frac{7}{4}x = \frac{4}{7} \cdot 35$$

Multiply each side by $\frac{4}{7}$.

$$x = 20$$

Simplify.

2. D

Let x and y be the two numbers.

$$x + y = 14$$

The sum of two numbers is 14.

$$\frac{x}{y} = -3$$

The ratio of the two numbers is -3 .

$$\frac{x}{y} = -3 \Rightarrow x = -3y$$

$$x + y = 14$$

First equation

$$(-3y) + y = 14$$

Substitute $-3y$ for x .

$$-2y = 14$$

Simplify.

$$y = -7$$

Substitute $y = -7$ in the first equation.

$$x + (-7) = 14 \Rightarrow x = 21$$

Therefore the product of the two numbers is

$$x \cdot y = 21 \cdot (-7) = -147.$$

3. C

$$2(x - y) = 3y$$

$$2x - 2y = 3y$$

Distributive property

$$2x = 5y$$

Add $2y$ to each side.

$$\frac{2x}{2} = \frac{5y}{2}$$

Divide each side by 2.

$$x = \frac{5}{2}y$$

Simplify.

$$\frac{x}{y} = \frac{\frac{5}{2}y}{y}$$

Divide each side by y .

$$\frac{x}{y} = \frac{5}{2}$$

Simplify.

4. D

Let $6x$ = the length and $7x$ = the width of the rectangle.

$$P = 2\ell + 2w$$

Perimeter of a rectangle.

$$78 = 2(6x) + 2(7x)$$

 $P = 78$, $\ell = 6x$, and $w = 7x$

$$78 = 26x$$

Simplify.

$$3 = x$$

Divide each side by 26.

Therefore, the length of the rectangle is $6 \cdot 3$ or 18, and the width of the rectangle is $7 \cdot 3$ or 21. The area of the rectangle is $18 \cdot 21$ or 378.

5. 23

$$\text{Gas Mileage} = \frac{\text{Number of Miles Traveled}}{\text{Number of Gallons of Gas Used}}$$

$$= \frac{218.5}{9.5} = 23$$

The car's gas mileage is 23 miles per gallon.

6. 5

$$\text{Unit Price} = \frac{\text{Price of Package}}{\text{Number of Units in the Package}}$$

$$= \frac{0.95}{20} = 0.0475$$

The unit price of the vitamin water to the nearest cent is 5.

7. 32.4

$$\text{Density} = \frac{\text{mass}}{\text{volume}}$$

$$\frac{2.7 \text{ grams}}{1 \text{ cm}^3} = \frac{x \text{ grams}}{12 \text{ cm}^3}$$

$$x = 2.7 \times 12 = 32.4 \text{ grams}$$

Section 6-2

1. D

Set up a proportion.

$$\frac{1 \text{ inch}}{5 \text{ miles}} = \frac{10 \text{ inches}}{x \text{ miles}} \Rightarrow x = 50 \text{ miles}$$

$$\frac{1 \text{ inch}}{5 \text{ miles}} = \frac{7.2 \text{ inches}}{y \text{ miles}} \Rightarrow y = 7.2 \times 5 = 36 \text{ miles}$$

The area of the state is 50×36 , or $1,800 \text{ mi}^2$.

2. B

Let the number of students = $27x$ and let the number of teachers = $2x$. Then, there will be $27x + 2x$, or $29x$ students and teachers who are in the meeting. Now set up a proportion.

$$\frac{\text{total in the meeting}}{\text{number of teachers}} = \frac{29x}{2x} \Rightarrow$$

$$\frac{754}{\text{number of teachers}} = \frac{29\cancel{x}}{2\cancel{x}} \Rightarrow$$

$$\text{number of teachers} = \frac{754 \times 2}{29} = 52$$

3. B

Let $5x$ = the volume of cement, $9x$ = the volume of sand, and $13x$ = the volume of gravel. Thus the total volume of concrete is $5x + 9x + 13x$, or $27x$, which is equal to 324 ft^3 .

$$27x = 324 \Rightarrow x = 12$$

Therefore, the amount of cement is $5x = 5 \cdot 12 = 60$.

4. 240

$$1 \text{ hour } 45 \text{ minutes} = 1\frac{3}{4} \text{ hours}$$

Set up a proportion.

$$\frac{84 \text{ miles}}{1\frac{3}{4} \text{ hours}} = \frac{x \text{ miles}}{5 \text{ hours}}$$

$$1\frac{3}{4}x = 84 \cdot 5 \quad \text{Cross Products}$$

$$\frac{7}{4}x = 420 \quad \text{Simplify.}$$

$$\frac{4}{7} \cdot \frac{7}{4}x = \frac{4}{7} \cdot 420 \quad \text{Multiply each side by } \frac{4}{7}.$$

$$x = 240$$

He can drive 240 miles in 5 hours.

5. 8

Let $2x$ = the number of quarters, $4x$ = the number of dimes, and $7x$ = the number of nickels.

Then the total amount in terms of x , $2x(0.25) + 4x(0.1) + 7x(0.05)$, is equal to \$5.00.

$$2x(0.25) + 4x(0.1) + 7x(0.05) = 5.00$$

$$0.5x + 0.4x + 0.35x = 5$$

$$1.25x = 5$$

$$x = 4$$

There are $2x = 2 \cdot 4$, or 8 quarters.

6. 6

$$\frac{5x}{3} = \frac{x+14}{2}$$

$$2(5x) = 3(x+14) \quad \text{Cross Products}$$

$$10x = 3x + 42$$

$$7x = 42$$

$$x = 6$$

7. $\frac{5}{12}$

Let r = the amount of raisin, p = the amount of peanut, and c = the amount of chocolate. Then

$$\frac{r}{p} = \frac{2}{3} \quad \text{The ratio of raisin to peanut is } 2:3.$$

$$3r = 2p \quad \text{Cross Products}$$

$$p = \frac{3}{2}r \quad \text{Solve for } p.$$

$$\frac{p}{c} = \frac{5}{8} \quad \text{The ratio of peanut to chocolate is } 5:8.$$

$$8p = 5c \quad \text{Cross Products}$$

$$p = \frac{5}{8}c \quad \text{Solve for } p.$$

Equate the two equations solved for p .

$$\frac{3}{2}r = \frac{5}{8}c \Rightarrow \frac{2}{3} \cdot \frac{3}{2}r = \frac{2}{3} \cdot \frac{5}{8}c$$

$$\Rightarrow r = \frac{5}{12}c \Rightarrow \frac{r}{c} = \frac{5}{12}$$