

SHSAT Lesson 3: Ratios and Proportions

In this lesson:

Definitions of a ratio and a proportion: comparing quantities with division.

Definition of a percentage: percentage as a kind of ratio.

What you must know

Definitions

A ratio is a way of comparing two numbers or quantities of items by using division. For example, consider the following set of triangles and circles:



In this set, the ratio of the number of triangles to the number of circles is 4 to 6, or 2 to 3 ($4/6 = 2/3$). One can also say that *for every 2 triangles, there are 3 circles*.

A proportion is a statement about an equivalency between two ratios. In the example above, the statement $2/3 = 4/6$ is a statement of proportionality. Proportions may also be expressed with unknown quantities, where you may be asked to find the unknown. For example: $x/3 = 6/9$.

A percentage is a ratio expressed in terms of fractions of 100. For example, 1% is $1/100$, 10% is $10/100$, and so on.

Sample Problems

1. A bag contains 3 red marbles, 11 yellow marbles, and 7 green marbles. What is the ratio of the number of green marbles to the number of non-green marbles in the bag?

A. 2:1 B. 7:13 C. 1:2 D. 7:11 E. 7:3

Solution

The total number of non-green marbles is $3 + 11 = 14$. Therefore, the ratio of the number of green to the number of non-green marbles is 7:14, or **C. 1:2** when simplified.

2. Which of the following statements of proportion is true?

A. $2:8 = 1:7$ B. $51:68 = 1:3$ C. $51:68 = 3:4$ D. $5:15 = 1:4$ E. $1:3 = 2:4$

Solution

Since $51 = 17 \cdot 3$ and $68 = 17 \cdot 4$, in the ratio 51:68, both sides can be divided by 17 to obtain an equivalent ratio of 3:4. Therefore, the statement of proportion **B. 51:68 = 1:3** is true.

3. The price of a shirt is lowered by 40%. Later, it is lowered again, by 10%. What single price reduction are these two price reductions equivalent to?

A. 0% B. 50% C. 30% D. 46% E. 54%

Solution

The easiest way to approach this problem is to assign a convenient price to the shirt. The easiest price to work with is \$100. A reduction of 40% from a price of \$100 is $\$100 - \$100 \cdot (40/100) = \$100 - \$40 = \$60$. A further reduction of 10% from that price is $\$60 - \$60 \cdot (10/100) = \$60 - \$6 = \$54$. Therefore, the difference between the original price of \$100 and the final price is $\$100 - \$54 = \$46$. This represents the total reduction in price, which is $\$46/\$100 = 46/100 = \mathbf{D. 46\%}$.

Practice Problems

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1. The height of a rectangle is twice its width. What is the ratio of the height of this rectangle to its perimeter?

A. 2 : 7
B. 7 : 12
C. 7 : 2
D. 1 : 3

2. If $a : b = 3 : 5$, what is the ratio of $10b$ to $6a$?

E. 25 : 9
F. 1 : 1
G. 5 : 3
H. 3 : 5

3. If $a : b$ is 1 : 2, and $b : c$ is 1 : 5, what is $a : c$?

A. 1 : 2
B. 2 : 5
C. 1 : 5
D. 1 : 10

4. If $1 : x$ is equal to $7 : 2$, then what is $1 : (x + 2)$?

E. 16 : 7
F. 7 : 16
G. 4 : 7
H. 9 : 2

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5. If the perimeter of a square is equal to the perimeter of an equilateral triangle, what is the ratio of the side of that square to the side of that triangle?

A. 4 : 7
B. 7 : 3
C. 4 : 3
D. 3 : 4

6. 2 mangos cost as much as 1 pear.
2 pears cost as much as 9 apples.
How many apples cost as much as 4 mangos?

E. 4
F. 6
G. 8
H. 9

7. Which ratio is equivalent to 15% of a number?

A. 4 : 15
B. 5 : 33
C. 3 : 25
D. 1.5 : 10

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8. There are 42 students in a class. Which of the following could be the ratio of the number of boys to the number of girls in this class?

E. 1 : 4
F. 2 : 13
G. 10 : 11
H. None of the above

9. If 12.7% of N is equal to 347, what is 1.27% of N ?

A. 0.347
B. 3.47
C. 34.7
D. 347

10. The ratio of the number of gray mice to the number of white mice in a cage is 3 : 5. When 3 more gray mice are added to the cage, the new ratio between numbers of gray and white mice becomes 3 : 4. How many mice were in the cage before 3 mice were added?

E. 16
F. 24
G. 32
H. 35

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11. Ben received 3% more votes than Sam during the team captain election. If Ben and Sam were the only two candidates, what percent of all votes did Sam receive?

A. 47%
B. 47.5%
C. 49%
D. 48.5%

12. The price of a coat increased by 20% in December, and then decreased by 20% three months later. If the final price of the coat was \$144, what was the original price of the coat before December?

E. \$120
F. \$150
G. \$160
H. \$144

13. Which number is 12% of one-half of 400?

A. 24
B. 25
C. 28
D. 96

14. In the pattern: 2, 4, 8, 16, 32, ..., what is the ratio of the 5th term to the 10th term?

- E.** 1 : 32
 - F.** 1 : 16
 - G.** 1 : 4
 - H.** None of the above
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15. Which of the following ratios is equivalent to 24%?

- A.** 24 : 50
 - B.** 6 : 35
 - C.** 3 : 25
 - D.** 12 : 50
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16. Which number is the opposite reciprocal of 0.5?

- E.** 0.5
 - F.** -5
 - G.** -2
 - H.** -0.5
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17. After one year, a \$1,200 deposit in a savings account earning a simple interest is worth \$1,218. Find the annual rate of interest for this account.

- A.** 2.4%
 - B.** 2%
 - C.** 1.8%
 - D.** 1.5%
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18. If one of two integers is 5% of the other, and their sum is 63, what is their product?

- E.** 360
 - F.** 180
 - G.** 320
 - H.** 163
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19. If you increase the radius of a circle by 50%, what percentage will the area of this circle increase by?

- A.** 124%
 - B.** 125%
 - C.** 120%
 - D.** 100%
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20. If the ratio of m to n is 7 to 13, then m is what percent of $(m + n)$?

- E.** 83.3%
 - F.** 80%
 - G.** 54%
 - H.** 35%
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21. What percentage of the numbers of the following set are prime?

{1, 2, 3, 4, 5, 6, 7, 8}

- A.** 75%
 - B.** 70%
 - C.** 62%
 - D.** 50%
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22. What percentage of all multiples of 3 smaller than 25 are prime?

- E.** 12.5%
 - F.** 25%
 - G.** 5%
 - H.** 0%
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23. 10% of 10% of N is what percentage of N ?

- A.** 0.01%
 - B.** 0.1%
 - C.** 1%
 - D.** 100%
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24. The number N was increased by 20%, and then the result was again increased by 20%. By what percent is the final result greater than the original number?

- E.** 44%
 - F.** 40%
 - G.** 20%
 - H.** 10%
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25. Which number in the following set is 15% of the sum of all numbers in the set: {7, 9, 11, 13, 20}?

- A.** 7
 - B.** 9
 - C.** 11
 - D.** 13
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