

SHSAT Lessons 5: Angles.

In this lesson:

Types of angles: supplementary and complementary angles, congruent angles, adjacent angles, vertical angles.

Angle formation: angles formed with two parallel lines and a transversal.

Angles in polygons: the sum of the angles of a triangle or any polygon, exterior angles.

Triangles: the rules for the sides, special triangles.

What you must know:

Definitions

Supplementary angles are two angles whose sum is equal to 180 degrees.

Complementary angles are two angles whose sum is equal to 90 degrees.

Congruent angles are two angles with the same measurement. **Adjacent angles** share a common vertex and side. **Vertical angles** are formed when two lines intersect.

Angles formed with two parallel lines and a transversal: if any transversal intersects two parallel lines, then any two acute angles formed at intersections are congruent; any two obtuse angles formed at intersections are congruent; and any one obtuse angle and any one acute angle formed at intersections are supplementary.

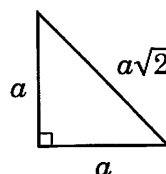
Interior and exterior angles of a triangle and any polygon: the sum of all interior angles of any triangle is 180° . The measure of any exterior angle of the triangle is equal to the sum of measures of two remote interior angles (interior angles that aren't adjacent to the given exterior angle).

For an n-sided polygon, the sum of all interior angles is $S(n) = 180^\circ(n - 2)$, where n is the number of sides. **The sum of all exterior angles** of any polygon (taking only one for each vertex) is 360° .

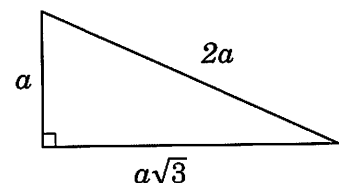
The rule for triangle sides: any side of any triangle is more than the difference and less than the sum of two other sides.

Special triangles' properties:

$45^\circ - 90^\circ - 45^\circ$:



$60^\circ - 90^\circ - 30^\circ$:



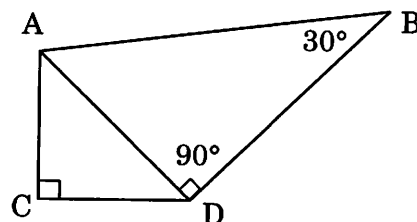
Sample Problems

1. If the sum of two exterior angles of a triangle is 300° , what is the largest possible measure of one of the interior angles of this triangle?

Solution

Since the sum of all exterior angles of any triangle is 360° , we can find the measure of the third exterior angle: $360^\circ - 300^\circ = 60^\circ$, and the interior angle which makes the linear pair with 60° is 120° ($180^\circ - 60^\circ = 120^\circ$). A triangle can have only one obtuse angle, so the largest possible measure of one of the interior angles of this triangle is **120°** .

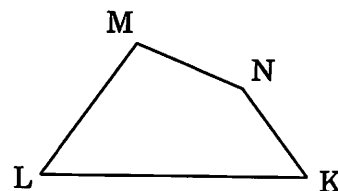
2. Two right triangles ACD and ADC share side AD . If $AC = DC$, and $AB = 8$, find the area of the triangle ACD .



Solution

Both triangles are special: ADB is 30° - 90° - 60° and ACD is 45° - 90° - 45° . AD is the leg opposite to 30° angle and AD is equal to one half of AB, thus $AD = 4$. $AC = DC = 2\sqrt{2}$ and the area of the triangle ACD = $(1/2)(2\sqrt{2})(2\sqrt{2}) = 4$.

3. In the quadrilateral $KLMN$, if $LM = 7$, $MN = 4$, $NK = 4$, and the perimeter of the $KLMN$ is an integer, what is the largest possible value of the perimeter?



Solution

KL < LM + MN + NK, so $KL < 7 + 4 + 4 \rightarrow KL < 15$. The largest possible integer value for KL is 14, and therefore the largest possible integral perimeter is $14 + 15 = 29$.

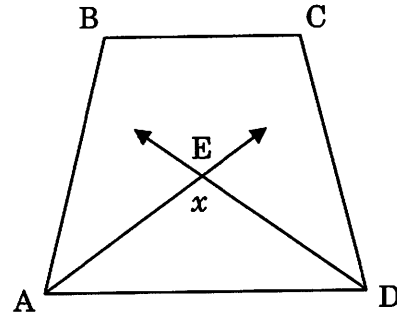
4. One of the base interior angles of an isosceles trapezoid is 70° . What is the measure of the larger angle between two angle bisectors of the base interior angles of the trapezoid?

Solution

We are given that the measure of the angle BAD and the measure of the angle CDA are 70° .

Bisectors AE and DE divide each angle into two equal parts of 35° .

Add all angles of triangle AED to get an equation: $35^\circ + x + 35^\circ = 180^\circ$, and find that $x = 110^\circ$. x is the larger of the two supplementary angles, so the answer is 110° .



5. Three interior angles of the triangle are consecutive multiples of 3. Find the smallest angle of the triangle.

Solution

Let the smallest interior angle be x , which means that the other two interior angles are $x + 3^\circ$ and $x + 6^\circ$ (consecutive multiples of 3 are 3 units apart). Now write the equation expressing the sum of all three interior angles: $x + (x + 3) + (x + 6) = 180^\circ$. $3x + 9 = 180^\circ$, and $x = 57^\circ$.

Practice Problems

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1. What is the difference between the sum of all interior angles of any decagon and the sum of all interior angles of any pentagon?

A. 540°
B. 720°
C. 900°
D. 1080°

2. If one of the angles of a triangle is equal to 110° , what is the average measure of the other two angles?

E. 35°
F. 36°
G. 38°
H. 55°

3. If one of the exterior angles of a triangle is equal to 142° , what is the sum of the two remote interior angles of this triangle?

A. 38°
B. 142°
C. 168°
D. 180°

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4. The complement of an angle is one-fifth of the measure of the angle itself. What is the measure of the angle?

E. 15°
F. 18°
G. 25°
H. 75°

5. What is the average of all exterior angles of the decagon?

A. 28°
B. 32°
C. 36°
D. 288°

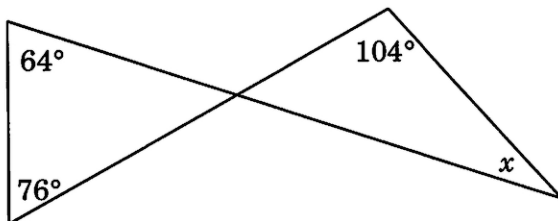
6. If two sides of triangle ABC are $AB = 12$ in and $BC = 7$ in, which value cannot represent the perimeter of this triangle?

E. 37 in
F. 33 in
G. 26 in
H. 24 in

7. What is the measure of an acute angle between two diagonals of the regular pentagon that originate from the same vertex?

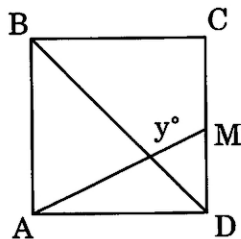
A. 24°
B. 26°
C. 28°
D. 36°

8. Find x in the diagram below.



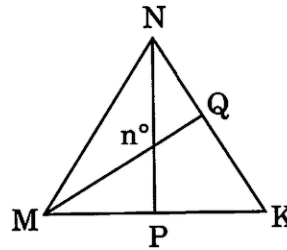
E. 16°
F. 28°
G. 36°
H. 66°

9. ABCD is a square. M is the midpoint of CD. Find the measure of angle y , if the measure of angle MAD is 32° .



A. 83°
B. 87°
C. 87°
D. 103°

10. In the equilateral triangle MNK, MQ is perpendicular to NK and NP is perpendicular to MK. What is the measure of angle n° ?



E. 110°
F. 120°
G. 130°
H. 140°

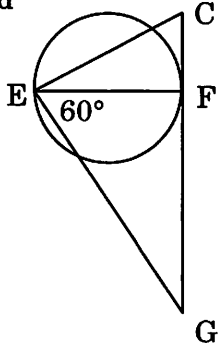
11. What is the measure of the angle between the hour and minute hands of a clock at 8:20 AM?

A. 90°
B. 100°
C. 110°
D. 130°

12. What is the difference between the average measure of all angles of a trapezoid and the measure of one angle of a rectangle?

E. 0°
F. 6°
G. 10°
H. 26°

13. EF is the diameter and CG is the tangent of a circle. If EG is 12 inches, what is the circumference of the circle?

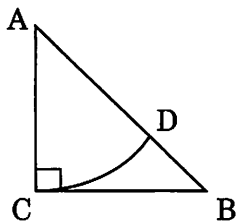


- A. 36π
- B. 12π
- C. 6π
- D. 3π

14. If the angle is 3 times bigger than its supplement, what is the difference between these two angles?

- E. 75°
- F. 60°
- G. 90°
- H. 135°

15. ABC is a right isosceles triangle and CD is an arc with the center at vertex A. If BC is 8 cm, what is the area of the sector (part of a circle) inside the triangle?

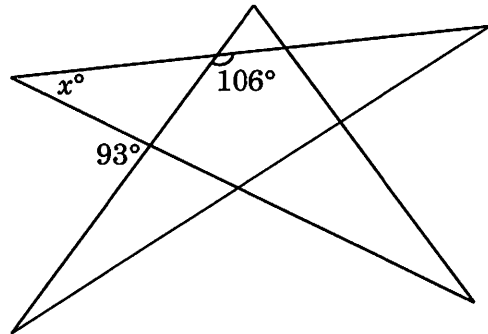


- A. 36π
- B. 8π
- C. 6π
- D. 3π

16. If the sum of two different exterior angles of the triangle is 266° , what is the measure of one of the interior angles of this triangle?

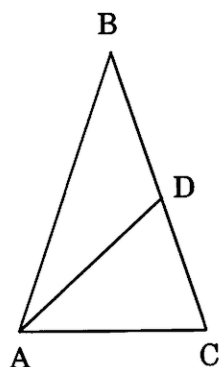
- E. 74°
- F. 84°
- G. 86°
- H. 106°

17. In the diagram below, find x .



- A. 19°
- B. 29°
- C. 41°
- D. 44°

18. In the isosceles triangle ABC, $AB = BC$, and $BD = AC = AD$. Find the measure of angle ABC.

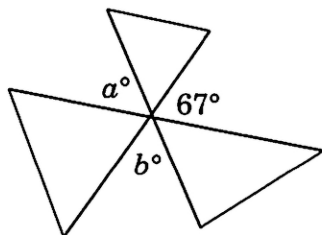


- E. 66°
F. 50°
G. 46°
H. 36°

19. Two sides of the triangle are 6 and 11. If each side has the integral measure, what is the largest possible perimeter of the triangle?

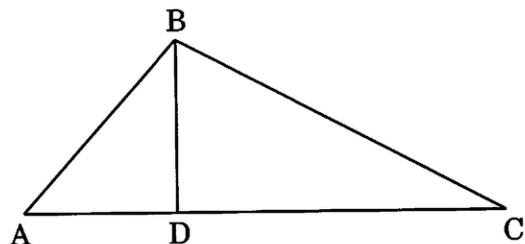
- A. 23
B. 27
C. 33
D. 37

20. Find the sum of $a^\circ + b^\circ$.



- E. 67°
F. 113°
G. 123°
H. 133°

21. In the diagram below, If BD is perpendicular to AC, $AD = BD$ and $BC = 2BD$, find the measure of angle ABC.



- A. 90°
B. 96°
C. 98°
D. 105°

22. Two bisectors of angles A and B of the triangle ABC intersect each other at point P. If the measure of angle C of the triangle is 86° , what is the measure of angle APB?

- E. 94°
F. 104°
G. 124°
H. 133°

23. Pentagon

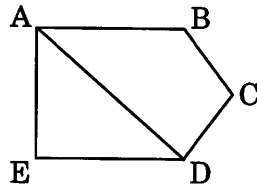
ABCDE

consists of the
square ABDE

and an

isosceles

triangle BCD. If the measure of
angle BCD is 112° , what is the
measure of angle ADC?



A. 93°

B. 79°

C. 76°

D. 74°

24. All sides of a triangle are integers.

If one side is 12, what is the
smallest possible difference between
the two other sides?

E. 15

F. 7

G. 1

H. 0

25. If the ratio of the three angles of the
triangle is $4 : 5 : 9$, what kind of the
triangle is it?

A. Isosceles

B. Right

C. Acute

D. Obtuse