

SHSAT Lesson 2: Patterns and Sequences

In this lesson:

Definitions of a pattern and a sequence: patterns of symbols and sequences of numbers.

Rules: Determining what comes next or what's missing in a pattern or sequence by figuring out what rules they follow.

Terminology: Mean (or average), median.

What you must know

Definitions

A pattern is a set of elements (numbers, symbols, or pictures) that are arranged following a particular rule. For example, consider the following pattern of numbers:

3, 10, 17, 24, 31, ...

This pattern follows this rule: "*start with 3, and add 7 for each subsequent element*". A pattern can also consist of non-numerical symbols. For example:

⌋ € € ⌋ ⌋ ⌋ € € ⌋ ⌋ ⌋ ⌋ € € ⌋ ⌋ ...

This pattern follows the rule: "*A group of symbols ⌋ € € ⌋ ⌋ repeats over and over*".

A sequence is an ordered collection of objects, with possible repetitions. Like any set, it contains members (also called elements, or terms). For example:

1. **1, 1, 2, 3, 5, 8, 13, ...** is the Fibonacci sequence. The first two terms are defined to be 1 and 1, and starting from the third term, each term is equal to the sum of two preceding terms.
2. **2, 5, 8, 11, 14, ...** is a sequence follows the rule $N = 3n - 1$, where N is the term and n is the position of the term in the sequence. Therefore, $N = 3n - 1$ is the formula to find the n^{th} term of the sequence.

An average (also known as a mean) is the sum of all the numbers in the set divided by how many of them are in the set.

A median is the middle number in an ordered set, if the set has an odd number of elements. If there is an even number of elements, a median is the average of the two middle numbers of the set.

Sample Problems

1. **15, 16, 23, 40, 51** is a set of 5 numbers. What is the average of the set?

Solution

The sum of the numbers is $(15 + 16 + 23 + 40 + 51) = 145$. There are five numbers in the set, so the average is $145 \div 5 = \mathbf{29}$.

2. If the average of 17 consecutive **odd** numbers is 29, what is the smallest number in the set?

Solution

When you are given a set of **consecutive numbers**, the average is the number that is in the middle of the set (if there is an odd number of terms) or the average of two middle terms (if there is an even number of terms). Therefore, 29 is the middle number of the set.

Since there are 17 numbers total, there must be 8 odd numbers before 29 and 8 odd numbers after 29. Since the difference between each two subsequent numbers in the set is 2, the smallest number in the set is 8 steps of size 2 below the middle number. We can write that as:

$$29 - 2 \cdot 8 = 13$$

Therefore, the smallest number of the set is 13.

Alternatively, you can simply write out the full set of numbers:

13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45

Practice Problems

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1. Tim received the average score of 82 for his first 4 tests in Science. Which score does he need to get for the 5th final test if he wants to have the average of 85?

A. 98
B. 97
C. 92
D. 88

2. The average of 10 integers is equal to -11 . What is the sum of these integers?

E. -11
F. -55
G. -110
H. 110

3. Find the 4th term of the sequence: $-2, 1, 7, _, 43, \dots$

A. 14
B. 19
C. 17
D. 15

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4. The average of the first 3 numbers of the set is 17 and the average of the next 5 numbers of the same set is 25. What is the average of the set of all 8 numbers?

E. 4
F. 11
G. 13
H. 22

5. The average of two multiples of 7 is 28. Find the larger number if it is 7 times greater than the smaller.

A. 28
B. 35
C. 42
D. 49

6. The sum of the first 9 smallest positive integers is not divisible by

E. 3
F. 9
G. 6
H. 15

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7. The product of the first 10 consecutive prime numbers is not divisible by

A. 25
B. 22
C. 6
D. 2

8. Each term of the sequence is the nonnegative difference of two preceding terms. Find the missing term: 15, 7, 8, 1, 7, 6, _, 5, 4, ...

E. 0
F. 1
G. 2
H. 7

9. In the repeating decimal 0.472819472819472819..., what is the 32nd digit after the decimal point?

A. 1
B. 2
C. 4
D. 7

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10. If the average of 6 distinct positive integers is 11, what is the largest possible integer could be one of these integers?

E. 61
F. 51
G. 41
H. 31

11. What is the 6th term of this pattern: 0.1, 0.02, 0.12, 0.14, 0.26, _, ...

A. 0.4
B. 0.3
C. 0.7
D. 0.9

12. Which expression can be used to find the value of the n^{th} term of the pattern described by the table below?

Term	1	2	3	4	5	6	...	n
Value	8	6	4	2	0	-2	...	?

E. $8n$
F. $2n + 2$
G. $10 - 2n$
H. $2n - 1$

13. What is the average of 4 smallest positive multiples of 4?

- A. 10
 - B. 8
 - C. 4
 - D. 2
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14. What is the 10th term of the following pattern: $2a + 3$, $4a + 6$, $6a + 9$, ...

- E. $2a - 10$
 - F. $12a + 15$
 - G. $16a + 18$
 - H. $20a + 30$
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15. Which number cannot be equal to the average of any two prime numbers?

- A. 15
 - B. 13.5
 - C. 13
 - D. 6.5
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16. What is the 100th term of this pattern:

A123B123C123A123B123C123A123...

- E. A
 - F. B
 - G. 2
 - H. 3
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17. What is the difference between the mean and the median of any 17 consecutive integers?

- A. 0
 - B. 0.5
 - C. 1
 - D. 2
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18. Which number cannot be the difference between two consecutive perfect squares?

- E. 17
 - F. 29
 - G. 41
 - H. 44
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19. If you rearrange the set of integers $\{-4, 27, -34, -23, 3, 1, 0\}$ in order from smallest to largest, which number will be in the middle?

- A. 1
 - B. -34
 - C. 3
 - D. 0
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20. What is the range of 23 consecutive multiples of 3?

- E. 27
 - F. 60
 - G. 66
 - H. 69
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21. If $A \& B = A^2 - B^3$, find $(-2) \& (-1)$.

- A. 5
 - B. -5
 - C. -4
 - D. 1
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22. What is the 20th decimal digit of the repeating fraction $3/7$?

- E. 4
 - F. 2
 - G. 5
 - H. 1
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23. Find the missing term of the pattern:

A, D, G, J, M, , S, V, Y

- A. N
- B. Q
- C. P
- D. R

24. Which rule for the n^{th} term describes the pattern: 97, 94, 91, 88, ... ?

- E. $3n$
 - F. $3n - 2$
 - G. $90n + 7$
 - H. $100 - 3n$
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25. If $a + b + c + d = 420$, what is the average of a , b , c , and d ?

- A. 210
- B. 105
- C. 21
- D. Cannot be determined