

SHSAT Lessons 6: Problem Solving Strategies.

In this lesson:

Graphic approach: visualizing with pictures, drawing diagrams.

Pattern recognition: repetitions, sequences, regularities.

Substitution: plugging in numbers, replacing variables with numbers.

Solving strategies: working backwards, solving a special case or a simpler problem, checking all possible cases, writing out an equation.

What you must know:

The more problem-solving strategies you know, the better equipped you'll be to answer SHSAT questions quickly and effectively. Often, it can help to use more than one solving strategy on any given problem. In the sample problems, we explore some of the strategies listed above.

Sample Problems

Drawing a Diagram

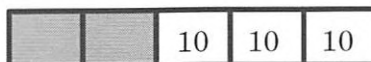
1. Max used $\frac{1}{5}$ th of his envelopes on Monday, and $\frac{1}{4}$ th of the remaining envelopes on Tuesday. How many envelopes did he have in the beginning, if he now has 30 envelopes left over?

Solution

Represent the total number of envelopes with a bar graph. Since you know that on Monday Max used $\frac{1}{5}$ th of the envelopes, divide the bar into 5 equal parts and mark one of them as used:



Now note that there are 4 parts left, and recall that Max next used $\frac{1}{4}$ th of the remaining envelopes. Therefore, you can mark one of the leftover parts as used:



Solution (continued)

Finally, note that the 3 parts left correspond to 30 envelopes. Therefore, each part in the graph above is equal to 10 envelopes. Since there were originally 5 parts, Max originally had $5 \times 10 = 50$ envelopes.

Recognizing a pattern in a sequence

2. If the given pattern continues, which fraction will be in the 30th position?

$$\frac{2}{5}, \frac{7}{10}, \frac{12}{15}, \frac{17}{20}, \frac{22}{25}, \dots$$

Solution

Observe the denominators and note that they are consecutive multiples of 5. Furthermore, note that in each fraction, the numerator is 3 less than the denominator. Therefore, the denominator of the 30th term will be equal to $5 \cdot 30 = 150$ and the numerator will be $150 - 3 = 147$. Thus, the 30th fraction in the pattern will be $\frac{147}{150}$.

Number substitution

3. What is the largest possible product of a and b , if $-8 \leq a \leq -2$ and $-4 \leq b \leq 1$?

Solution

Among the possible values for a and b , choose the ones with the greatest magnitude (absolute value) – since the product of two negative numbers is positive. $ab = (-8)(-4) = 32$.

Observing regularities

4. Find the two missing numbers from the table:

10	15	20	25	30	35	40	45	50	55
1	6	2	7	3	8	?	9	5	?

Solution

Observe the two numbers in each column, and note that each number in the second row is the sum of the digits of the number in the first row. Therefore, the missing numbers are $4 + 0 = 4$, and $5 + 5 = 10$.

Working backwards

5. Sam chose a positive integer less than 20. He multiplied the chosen integer by 6 and then divided it by 2. He then divided the result by 3 and multiplied it by 5. If at the end he got 35, find the integer he started with.

Solution

Suppose Sam chose the number N . Combine the listed operations and write them as an equation: $N \cdot 6 \cdot \frac{1}{2} \cdot \frac{1}{3} \cdot 5 = 5N = 35$, and $N = 7$.

Or work backwards and perform the listed operations in an opposite order:

$$35 \div 5 = 7 \rightarrow 7 \cdot 3 = 21 \rightarrow 21 \cdot 2 = 42 \rightarrow 42 \div 6 = 7.$$

Writing out an equation

6. If I had twice the amount of money I have now, I could buy 4 lollipops for 75 cents each and get \$1 change. How much money do I have now?

Solution

Write the verbal description in the problem as an equation. Calling the amount of money I have x , “twice the amount of money I have now” is $2x$. “4 lollipops for 75 cents each” is a dollar quantity of $4 \cdot \$0.75$, and there is also a \$1 change, which means that we can write $2x = 4 \cdot \$0.75 + \1 . Therefore, $2x = \$4$, and $x = \$2$.

Practice Problems

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1. James solved $\frac{1}{4}^{\text{th}}$ of his homework problems in 15 minutes. In the next 15 minutes, he solved $\frac{1}{3}^{\text{rd}}$ of the remaining problems. If James has 12 more problems left after that, how many problems were there in his homework?
- A. 40
B. 36
C. 30
D. 24
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2. Mrs. Wagner spent one-third of her money in one store, and then one-half of the remaining money in another store. How much money did Mrs. Wagner have in the beginning, if she returned home with \$17?
- E. \$68
F. \$64
G. \$51
H. \$17
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3. The perimeter of a rectangle is 6 times as large as the width of the rectangle. How many times is the perimeter of this rectangle larger than its length?
- A. $\frac{1}{3}$
B. $\frac{1}{2}$
C. 2
D. 3
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4. If k is an even number, what is the remainder when $(k + 2)^2$ is divided by 4?
- E. 0
F. 1
G. 2
H. 3
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5. The number of bacteria in a bacterial culture doubles every 10 minutes. The ratio of the number of bacteria at the end of 1 hour to the number of bacteria at the beginning of that hour is:
- A. 2 : 1
B. 8 : 1
C. 64 : 1
D. 1024 : 1
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6. If a set of numbers begins with -11 and every number after that is 10 more the number that comes before it, what is the 100^{th} number in the set?
- E. 959
F. 969
G. 979
H. 989
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7. The product of the first 50 prime numbers is not divisible by:

A. 26
B. 36
C. 46
D. 86

8. Find A, if $A - \frac{1}{2} - \frac{1}{4} = 0$.

E. 0.625
F. 0.65
G. 0.75
H. -0.75

9. Simplify:

$$(t + 1) - (t - 3) + (t + 5) - (t - 7) + \dots \\ \dots + (t + 17) - (t - 19) =$$

A. $19t - 100$
B. $t + 190$
C. $t - 190$
D. 100

10. If p and q are integers, and $p^2 = 1$, $q^2 = 81$, what is the smallest possible value of $(q - p)$?

E. -10
F. -8
G. 0
H. -1

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11. If the numerator of a fraction is 1 greater than its denominator, but smaller than 20, which of these is the largest possible value of this fraction?

A. $\frac{3}{2}$
B. $\frac{4}{3}$
C. $\frac{11}{10}$
D. $\frac{21}{20}$

12. For how many different integer values of N is $\frac{6}{N}$ also an integer?

E. 8
F. 7
G. 5
H. 4

13. Steve entered an elevator on a certain floor. Then the elevator moved up 7 floors, down 8 floors, and up 3 floors. On what floor did Steve initially enter the elevator, if he is now at floor 10?

A. 9
B. 8
C. 6
D. 4

14. I began with my favorite number, subtracted 7 from it, multiplied the result by 3, and then added 2 to that. My final result is 20. What is my favorite number?

- E.** 17
 - F.** 13
 - G.** 5
 - H.** 1
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15. Ann had a roll of fabric. She used $\frac{1}{2}$ of the roll, then $\frac{1}{2}$ of the remaining amount, and then $\frac{1}{2}$ of the remaining amount again. She now has 7 yards left over. How many yards of fabric did she have in the beginning?

- A.** 64 yards
 - B.** 60 yards
 - C.** 56 yards
 - D.** 40 yards
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16. The 3-digit number 103 has a digit-sum of $1 + 0 + 3 = 4$. How many different 3-digit whole numbers, including 103, have a digit sum of 4?

- E.** 6
- F.** 7
- G.** 9
- H.** 10

17. Find the largest factor of 1260 that is not divisible by 6.

- A.** 630
 - B.** 315
 - C.** 210
 - D.** 63
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18. When a certain number is divided by 5, the result is the same as when this number decreased by 12. What is this number?

- E.** 20
 - F.** 15
 - G.** 5
 - H.** -15
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19. Kathy has \$13 more than Jack and \$3 less than Vanessa. All together, Kathy, Jack, and Vanessa have \$59. How much money does Jack have?

- A.** \$10
- B.** \$12
- C.** \$18
- D.** \$7

20. Tickets for a play cost \$6 for a child, and \$15 for an adult. A group of 25 people paid a total of \$204 for the play. How many children were in this group?

- E. 20
- F. 18
- G. 12
- H. 19

21. If $a^a = |a| - \frac{1}{a}$, what is the value of $a^{(-2)^a}$?

- A. -2
- B. -1
- C. 0
- D. 2.5

22. If $b^\diamond = -(5b - 3)$, what is the value of $(-7)^\diamond$?

- E. 38
- F. -7
- G. 13
- H. -38

23. If $-m - n = 5$ and $m - n = -3$, what is the value of $m^2 - n^2$?

- A. 2
- B. -2
- C. 15
- D. -15

24. If the length of a rectangle is multiplied by 3, and the width of a rectangle is multiplied by 2, the area of the rectangle will be N times bigger. What is N?

- E. 6
- F. 3
- G. 2
- H. Cannot be determined

25. The price for a coat was reduced 3 times, each time by 20% from the price that preceded it. What is the total percentage of price reduction from the original price of the coat?

- A. 60%
- B. 51.2%
- C. 48.8%
- D. Cannot be determined